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A Multiobjective Fuzzy Linear Programming Problem Using Ranking Functions of Symmetric Trapezoidal Fuzzy Numbers

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Abstract: In this paper, a multi objective fuzzy linear programming problem with symmetric trapezoidal fuzzy numbers in which fuzzy parameters are used in both the objective functions and the constraints is considered. It is solved by using ranking functions of symmetric trapezoidal fuzzy numbers. The solution procedure is verified by means of a numerical example.Some concluding remarks are provided at the last.

Keywords: Multi objective fuzzy linear programming, Symmetric trapezoidal fuzzy numbers, ranking function.

1 Introduction

The fuzzy set theory is being applied in many fields these days. One of these is Linear programming problems. There are two kinds of linear programming problems, one is the linear programming with constraint conditions and the other is the linear programming with fuzzy coefficients. The latter may be divided in to three kinds as follows : constraint conditions with fuzzy coefficients, the object with fuzzy coefficients and the constraints & the object with fuzzy coefficients. The Simplex method for fuzzy variable linear programming problem discussed for single objective by [3] has been modified for multi objective problems as given in [4].In this Paper, we discuss a multi objective linear programming model as given in [4], in which the fuzzy parameters are involved in both the objective function and also the constraints. Moreover, symmetric trapezoidal fuzzy numbers and the ranking procedure given by [1] are used. Multi objective linear programming problem is the process of simultaneously optimizing two or more objective functions subject to certain constraints. In many real world problems, there are situations where multiple objectives may be more appropriate rather than considering single objective. The paper has the following structure. In section 2, symmetric trapezoidal fuzzy numbers , ranking function and arithmetic operations of symmetric trapezoidal fuzzy numbers are discussed as in [1]. Section 3 deals with multi objective fuzzy linear programming problem. In section 5, a numerical example is provided to illustrate its feasibility. The last section draws some concluding remarks.

2 PRELIMINARIES

In this section, we discuss the symmetric trapezoidal fuzzy numbers and their arithmetic operations as in [1].

2.1: Symmetric trapezoidal fuzzy number

Let us consider a symmetric trapezoidal fuzzy number $\tilde{a} = (a_1, a_2, h, h)$ whose membership function is given by

$$\tilde{a}(x) = \begin{cases} \dfrac{x}{h} + \dfrac{h - a_1}{h}, & x \in [a_1 - h, a_1] \\ 1, & x \in [a_1, a_2] \\ \dfrac{-x}{h} + \dfrac{a_2 + h}{h}, & x \in [a_2, a_2 + h] \\ 0, & \text{otherwise} \end{cases}$$

where $a_1 \le a_2$ and $h \ge 0$ in the real line R.

2.2 : Ranking function

Let $\mathcal{F}$ (S) be the set of all symmetric trapezoidal fuzzy numbers.
For $\tilde{a} = (a_1, a_2, h, h) \in \mathcal{F}(S)$, we define a ranking function
F : $\mathcal{F}(S) \to R$ by

$$F(\tilde{a}) = \frac{(a_1 - h) + (a_2 + h)}{2} = \frac{a_1 + a_2}{2}$$ as in [1]

2.3: Arithmetic operations on symmetric trapezoidal fuzzy numbers

For $\tilde{x} = (x_1, x_2, h, h)$ and $\tilde{y} = (y_1, y_2, k, k)$ in $\mathcal{F}$(S), we define

Addition : $\tilde{x} + \tilde{y} = (x_1, x_2, h, h) + (y_1, y_2, k, k)$

$= ((F(\tilde{x}) + F(\tilde{y})) - s, \ (F(\tilde{x}) + F(\tilde{y})) + s, \ h + k, h + k)$

where $s = \dfrac{(y_2 + x_2) - (y_1 + x_1)}{2}$

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Subtraction: $\tilde{x} - \tilde{y} = (x_1, x_2, h, h) - (y_1, y_2, k, k)$

$= ((F(\tilde{x}) - F(\tilde{y})) - s,\ (F(\tilde{x}) - F(\tilde{y})) + s,\ h + k, h + k)$

where $s = \dfrac{(y_2 + x_2) - (y_1 + x_1)}{2}$

Multiplication: $\tilde{x}\tilde{y} = (x_1, x_2, h, h)\ (y_1, y_2, k, k)$

$= ((F(\tilde{x})\ F(\tilde{y})) - s,\ (F(\tilde{x})\ F(\tilde{y})) + s,\ |x_2 h + y_2 k|,\ |x_2 h + y_2 k|)$

where $s = \dfrac{\beta - \alpha}{2}$

Division: $\dfrac{1}{(x_1, x_2, h, h)} = \left[\dfrac{1}{F(\tilde{x})} - s, \dfrac{1}{F(\tilde{x})} + s, h, h\right]$

where **s =** $\dfrac{1}{2}\left(\dfrac{1}{x_1} - \dfrac{1}{x_2}\right)$

Scalar multiplication:

$$\lambda \tilde{x} = \begin{cases} (\lambda x_1, \lambda x_2, \lambda h, \lambda h), & \text{for } \lambda \geq 0 \\ (\lambda x_2, \lambda x_1, -\lambda h, -\lambda h), & \text{for } \lambda < 0 \end{cases}$$

3 MULTI OBJECTIVE FUZZY LINEAR PROGRAMMING PROBLEM

The multi objective fuzzy linear programming problem can be formulated as follows

$$\text{Max } \sum_{j=1}^{n} \tilde{c}_{kj}\tilde{x}_j \qquad k = 1,2,\ldots..p$$

Subject to the constraints

$$\sum \tilde{a}_{ij}\tilde{x}_j \leq \tilde{b}_i \qquad i = 1,2\ldots\ldots m$$

where $\tilde{c}_{kj}, \tilde{a}_{ij}, \tilde{b}_i$ are fuzzy numbers.

4 NUMERICAL EXAMPLE

We consider a Multi objective fuzzy linear programming problem

$$\text{Max } Z_1 = \tilde{5}\ \tilde{x}_1 + \tilde{x}_2$$
$$\text{Max } Z_2 = \tilde{x}_1 + \tilde{x}_2$$
$$\text{Subject to } \tilde{x}_1 + \tilde{x}_2 \leq \tilde{6}$$
$$\tilde{x}_1 \leq \tilde{5}$$
$$\tilde{x}_1, \tilde{x}_2 \geq 0$$

First we transform all the fuzzy coefficients in to symmetric trapezoidal fuzzy numbers

$$\text{Max } Z_1 = (4, 6, 2, 2)\,\tilde{x}_1 + (0.75,1.25, 0.5,0.5)\,\tilde{x}_2$$

$$\text{Max } Z_2 = (0.75,1.25, 0.5,0.5)\,\tilde{x}_1 + (0.75,1.25, 0.5,0.5)\,\tilde{x}_2$$

Subject to the constraints

$$(0.75,1.25, 0.5,0.5)\,\tilde{x}_1 + (0.75,1.25, 0.5,0.5)\,\tilde{x}_2 \leq (5, 7, 2, 2)$$

$$(0.75,1.25, 0.5,0.5)\,\tilde{x}_1 \leq (4, 6, 2, 2)$$

$$\tilde{x}_1, \tilde{x}_2 \geq 0$$

Now, we solve

$$\text{Max } Z_1 = (4, 6, 2, 2)\,\tilde{x}_1 + (0.75,1.25, 0.5,0.5)\,\tilde{x}_2$$

Subject to the constraints

$$(0.75,1.25, 0.5,0.5)\,\tilde{x}_1 + (0.75,1.25, 0.5,0.5)\,\tilde{x}_2 \leq (5, 7, 2, 2)$$

$$(0.75,1.25, 0.5,0.5)\,\tilde{x}_1 \leq (4, 6, 2, 2)$$

$$\tilde{x}_1, \tilde{x}_2 \geq 0$$

we rewrite the above problem in the standard form

$$\text{Max } Z_1 = (4, 6, 2, 2)\,\tilde{x}_1 + (0.75,1.25, 0.5,0.5)\,\tilde{x}_2$$

Subject to the constraints

$$(0.75,1.25,0.5,0.5)\,\tilde{x}_1 + (0.75,1.25, 0.5,0.5)\,\tilde{x}_2 + (0.75,1.25, 0.5,0.5)\,\tilde{x}_3 = (5, 7, 2, 2)$$

$$(0.75,1.25, 0.5,0.5)\,\tilde{x}_1 + (0.75,1.25, 0.5,0.5)\,\tilde{x}_4 = (4, 6, 2, 2)$$

$$\tilde{x}_1, \tilde{x}_2, \tilde{x}_3, \tilde{x}_4 \geq 0$$

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		(4,6,2,2)	(0.75, 1.25, 0.5,0.5)	(0,0,0,0)	(0,0,0,0		
C_j	B	$\tilde{x}_1$	$\tilde{x}_2$	$\tilde{x}_3$	$\tilde{x}_4$	RHS	F
(0,0,0,0)	$\tilde{x}_3$	(0.75,1.25, 0.5,0.5)	(0.75,1.25, 0.5,0.5)	(0.75,1.25, 0.5,0.5)	(0,0,0,0)	(5,7,2,2)	6
(0,0,0,0)	$\tilde{x}_4$	**(0.75,1.25, 0.5,0.5)**	(0,0,0,0)	(0,0,0,0)	(0.75,1.25, 0.5,0.5)	(4,6,2,2)	**5**
	$Z_j - C_j$	**(-6,-4,2,2)**	(-1.25, -0.75,0.5, 0.5)	(0,0,0,0)	(0,0,0,0)		
(0,0,0,0)	$\tilde{x}_3$	(-0.44, 0.44,1.76, 1.76)	**(0.75,1.25, 0.5,0.5)**	(0.75,1.25, 0.5,0.5)	(-0.44, 0.44,1.76, 1.76)	(-2.25, 4.25,7.5, 7.5)	**1**
(4,6,2,2)	$\tilde{x}_1$	(0.75,1.25, 0.5,0.5)	(0,0,0,0)	(0,0,0,0)	(0.75,1.25, 0.5,0.5)	(4,6,2,2)	5
	$Z_j - C_j$	(-3.25, 3.25,7.5, 7.5)	**(-1.25, -0.75, 0.5.0.5)**	(0,0,0,0)	(2.75,7.25, 5.5,5.5)		
(0.75, 1.25, 0.5, 0.5)	$\tilde{x}_2$	(-0.44, 0.44,1.76, 1.76)	(0.75,1.25, 0.5,0.5)	(0.75,1.25, 0.5,0.5)	(-0.44, 0.44,1.76, 1.76)	(-2.25, 4.25,7.5, 7.5)	
(4,6,2,2)	$\tilde{x}_1$	(0.75,1.25, 0.5,0.5)	(0,0,0,0)	(0,0,0,0)	(0.75,1.25, 0.5,0.5)	(4,6,2,2)	
	$Z_j - C_j$	(-3.80, 3.80,9.92, 9.92)	(-0.44, 0.44,1.76, 1.76)	(0.81, 1.19,1.26, 1.26)	(-0.55, 0.55,2.42, 2.42)		

Since all $Z_j - C_j \geq 0$, the optimal solution is obtained at

$\tilde{x}_1$=(4, 6, 2, 2) and $\tilde{x}_2$ = (-2.25, 4.25, 7.5, 7.5)

Hence, Max Z_1 = (12,40,35.51,35.51) , $F(Z_1)$ = 26.

Next, we solve

Max $Z_2 = (0.75,1.25, 0.5,0.5)\, \tilde{x}_1 + (0.75,1.25, 0.5,0.5)\, \tilde{x}_2$

Subject to the constraints

$$(0.75,1.25, 0.5,0.5)\, \tilde{x}_1 + (0.75,1.25, 0.5,0.5)\, \tilde{x}_2 \leq (5, 7, 2, 2)$$

$$(0.75,1.25, 0.5,0.5)\, \tilde{x}_1 \leq (4, 6, 2, 2)$$

$$(4, 6, 2, 2)\, \tilde{x}_1 + (0.75, 1.25, 0.5, 0.5)\, \tilde{x}_2 \geq (12, 40, 35.51, 35.51)$$

$$\tilde{x}_1 ,\ \tilde{x}_2 \geq 0$$

Now , we rewrite the above problem in the standard form

Max $Z_2 = (0.75,1.25, 0.5,0.5)\, \tilde{x}_1$ +(0.75,1.25, 0.5,0.5)

Subject to the constraints

$$(0.75,1.25, 0.5,0.5)\, \tilde{x}_1 + (0.75,1.25, 0.5,0.5)\, \tilde{x}_2 + (0.75,1.25, 0.5,0.5)\, \tilde{x}_3 = (5, 7, 2, 2)$$

$$(0.75,1.25, 0.5,0.5)\, \tilde{x}_1 + (0.75,1.25, 0.5,0.5)\, \tilde{x}_4 = (4, 6, 2, 2)$$

$$(-6,-4,2,2)\, \tilde{x}_1 + (-1.25,-0.75,0.5,0.5)\, \tilde{x}_2 + (0.75,1.25, 0.5,0.5)\, \tilde{x}_5 = (-40,-12 ,35.51,35.51)$$

$$\tilde{x}_1 ,\ \tilde{x}_2 ,\ \tilde{x}_3, \tilde{x}_4 ,\ \tilde{x}_5 \geq 0.$$

C_j	B	$\tilde{x}_1$	$\tilde{x}_2$	$\tilde{x}_3$	$\tilde{x}_4$	$\tilde{x}_5$	RHS	F
(0,0,0,0)	$\tilde{x}_3$	(0.75, 1.25, 0.5,0.5)	(0.75, 1.25, 0.5,0.5)	(0.75, 1.25, 0.5,0.5)	(0,0,0,0)	(0,0,0,0)	(5,7,2,2)	6
(0,0,0,0)	$\tilde{x}_4$	**(0.75, 1.25, 0.5,0.5)**	(0,0,0,0)	(0,0,0,0)	(0.75, 1.25, 0.5,0.5)	(0,0,0,0)	(4,6,2,2)	**5**
(0,0,0,0)	$\tilde{x}_5$	(-6,-4, 2,2)	(-1.25, -0.75, 0.5,0.5)	(0,0, 0,0)	(0,0, 0,0)	(0.75, 1.25, 0.5, 0.5)	(-40, -12, 35.51, 35.51)	-26
	$Z_j - C_j$	**(-1.25, -0.75, 0.5,0.5)**	(-1.25, -0.75, 0.5,0.5)	(0,0, 0,0)	(0,0, 0,0)	(0,0, 0,0)		
(0,0,0,0)	$\tilde{x}_3$	(-0.44, 0.44, 1.76, 1.76)	**(0.75, 1.25, 0.5,0.5)**	(0.75, 1.25, 0.5,0.5)	(-1.19, -0.81, 1.26, 1.26)	(0,0, 0,0)	(-2.25, 4.25, 7.5, 7.5)	**1**
(0.75, 1.25, 0.5, 0.5)	$\tilde{x}_1$	(0.75, 1.25, 0.5,0.5)	(0,0, 0,0)	(0,0, 0,0)	(0.75, 1.25, 0.5,0.5)	(0,0, 0,0)	(4,6, 2,2)	5
(0,0,0, 0)	$\tilde{x}_5$	(-3.5, 3.5, 2.25, 2.25)	(-1.25,- 0.75, 0.5,0.5)	(0,0, 0,0)	(2.25, 7.25, 0.25,0. 25)	(0.75, 1.25, 0.5,0. 5)	(-25, 23, 39.51, 39.51)	-1
	$Z_j - C_j$	(-0.44, 0.44, 1.76, 1.76)	**(-1.25, -0.75, 0.5,0.5)**	(0,0, 0,0)	(0.81, 1.19, 1.26, 1.26)	(0,0, 0,0)		
(0.75, 1.25, 0.5, 0.5)	$\tilde{x}_2$	(-0.44, 0.44, 1.76, 1.76)	(0.75, 1.25, 0.5,0.5)	(0.75, 1.25, 0.5,0.5)	(-1.19, -0.81, 1.26,1. 26)	(0,0, 0,0)	(-2.25, 4.25, 7.5,7.5)	
(0.75, 1.25, 0.5, 0.5)	$\tilde{x}_1$	(0.75, 1.25, 0.5,0.5)	(0,0, 0,0)	(0,0, 0,0)	(0.75, 1.25, 0.5,0.5)	(0,0, 0,0)	(4,6, 2,2)	
(0,0,0, 0)	$\tilde{x}_5$	(-4.05, 4.05, 3.37, 3.37)	(-0.75, 0.75, 0.75, 0.75)	(0.5, 1.5, 0.25, 0.25)	(2.95, 5.05, 1.61, 1.61)	(0.75, 1.25, 0.5,0. 5)	(-28.06, 28.06, 43.01, 43.01)	
	$Z_j - C_j$	(0.26, 1.74, 3.68, 3.68)	(-0.44, 0.44, 1.76, 1.76) 76)	(0.81, 1.19, 1.26, 1.26)	(-0.63, 0.63, 2.43,2. 43)	(0,0,0, 0)		

Since all $Z_j - C_j \geq 0$, the optimal solution is obtained at

$\tilde{x}_1$ =(4, 6, 2, 2) and $\tilde{x}_2$ = (-2.25, 4.25, 7.5, 7.5)

Hence, Max Z_2 =(1.75,10.25,9.5,9.5) , $F(Z_2) = 6$.

CONCLUSION

In this paper, we have considered a Multi objective fuzzy Linear programming problem with all coefficients and variables of the objective function and constraints are considered as symmetric trapezoidal fuzzy numbers. The optimal solution is obtained by using the ranking functions as in [1]. A numerical example is solved by using the preemptive optimization method and got the same result as the crisp linear programming problem.

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